THE CHINESE UNIVERSITY OF HONG KONG
DEPARTMENT OF MATHEMATICS
MATH2010D Advanced Calculus 2019-2020

Problem Set 10

1. (a) Find the absolute maximum and minimum values of the function $f(x, y) = xy$ subject the the constraint

$$\frac{x^2}{8} + \frac{y^2}{2} = 1.$$

   (b) In fact, the constraint in part (a) defines an ellipse which can be parametrized as $\gamma(t) = (2\sqrt{2}\cos t, \sqrt{2}\sin t)$, where $0 \leq t \leq 2\pi$.

   Therefore, the question in part (a) is equivalent to finding absolute extrema of the single variable function $f(\gamma(t))$ (by abuse of notation, it is simply denoted by $f(t)$).

   Using techniques in single variable calculus to find absolute extrema of $f(t)$ and verify the answer in (a).

2. Find the maximum and minimum values of the function $f(x, y, z) = 4x - 7y + 6z$ subjected to the constraint $g(x, y, z) = x^2 + 7y^2 + 12z^2 = 104$.

3. Let $f(w, x, y, z) = \left(w + \frac{1}{w}\right)^2 + \left(x + \frac{1}{x}\right)^2 + \left(y + \frac{1}{y}\right)^2 + \left(z + \frac{1}{z}\right)^2$ for $w, x, y, z > 0$.

   Prove that f is bounded below by $\frac{289}{4}$ on the plane $w + x + y + z = 16$.

4. Find the absolute maximum and minimum values of the function $f(x, y, z) = x$ over the curve of intersection of the plane $z = x + y$ and the ellipsoid $x^2 + 2y^2 + 2z^2 = 8$.